Code No: 114DD

R13

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
B. Tech II Year II Semester Examinations, September/October - 2023
MATHEMATICS - II
(Common to ME, MIE)

Time: 3 hours **Max. Marks: 75**

Note: i) Question paper consists of Part A, Part B.
ii) Part A is compulsory, which carries 25 marks. In Part A, Answer all questions.
iii) In Part B, Answer any one question from each unit. Each question carries 10 marks and may have a, b as sub questions.

PART - A

(25 Marks)

1.a) Calculate the Curl of the vector $xyz\, i + 3x^2y\, j + (xz^2 - y^2z)k$. [2]
b) Is the gradient describes the motion of a fluid as Irrotatinal? Explain. [3]
c) Find the constant term in the Fourier series expansion for the function $f(x) = x$ in $(0,2\pi)$. [2]
d) State the change of scale property for the Fourier Transform and prove it. [3]
e) Write the geometrical interpretation of the Newton-Raphson method. [2]
f) Find the interpolating polynomial for the following data: [3]

x	0	1	2	5
$f(x)$	2	3	12	147

g) Explain the regula-falsi method. [2]
h) Write the Newton-Raphson procedure to find $\sqrt[3]{N}$. [3]
i) State the Simpson's 3/8 rule. [2]
j) Estimate the integral $\int_0^{\frac{1}{2}} \frac{dx}{\sqrt{x}\sqrt{1-x}}$ using the trapezoidal rule. [3]

PART - B

(50 Marks)

2.a) Prove that $\nabla(f^n) = nf^{n-1}\nabla f$.
b) Evaluate $\oint_c \overline{F}.d\overline{r}$ around the boundary C of the region R by Green's Theorem, where,

$F = x^2y^2\overline{i} - \frac{x}{y^2}\overline{j}$, $R: 1 \le x^2 + y^2 \le 4, x \ge 0, y \ge x$. [5+5]

OR

3.a) At what point $P(x, y, z)$ does the directional derivative of $f = 25x^2 + 9y^2 + 16\, z^2$ be in the direction from P to the origin?
b) Evaluate $\int_S \overline{F}.\overline{n}\, ds$,where $\overline{F} = (0, \sinh z, \cosh x)$ and $S: x^2 + z^2 = 4,\ 0 \le x \le \frac{1}{\sqrt{2}},\ 0 \le y \le 5,\ z \ge 0$

[5+5]

4.a) Is $f(x)=\begin{cases} -\frac{1}{2}(\pi+x) & for\ -\pi \le x < 0 \\ \frac{1}{2}(\pi-x) & for\ 0 < x \le \pi \end{cases}$, and $f(x+2\pi)=f(x)\forall x\in R$ even? If so, find the Fourier series for the function. Deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots\ldots=\frac{\pi^2}{8}$

b) Find the Fourier series for $f(x)=2lx-x^2$ in $0<x<2l$. [5+5]

OR

5.a) Find the Fourier transform of $f(x)=\begin{cases} a^2-x^2 & if\ |x|<a \\ 0 & if\ |x|>a \end{cases}$. Hence show that $\int_0^\infty \frac{\sin x-\cos x}{x^3}dx=\frac{\pi}{4}$.

b) Find the inverse Fourier sine transform of $F_s\{p\}=\frac{p}{1+p^2}$. [5+5]

6.a) Construct the Newton's forward interpolation polynomial for the following data:

x:	4	6	8	10
y:	1	3	8	16

Hence evaluate y for $x=5$.

b) Evaluate (i) $\Delta^2 \sin(px+q)$, (ii) Prove that $y_3=y_2+\Delta y_1+\Delta^2 y_0+\Delta^3 y_0$. [5+5]

OR

7.a) Prove that the identity $u_1x+u_2x^2+u_3x^3\ldots=\left(\frac{x}{1-x}\right)u_1+\left(\frac{x}{1-x}\right)^2\Delta u_1+\left(\frac{x}{1-x}\right)^3\Delta^2 u_1+\ldots$.

b) Fit a curve of the form $y=ax^2$ from the following data:

x	2.0	2.3	2.6	2.9	3.2
y	5.1	7.5	10.6	14.4	19.0

[5+5]

8.a) Find the real root of the equation $xe^x=2$ by the method of false position.

b) By using Newton Raphson method, find the real root of the equation $\cos x-x^2-x=0$. [5+5]

OR

9.a) Apply the iteration method to find the real roots of $e^{-x}-\sin x=0$, correct to three decimal places.

b) Derive a formula to find a k^{th} root of a number N using Newton–Raphson method. Find the real root of the equation $\tan x=1.2x$ correct to three decimal places. [5+5]

10.a) If $\frac{dy}{dx}=\frac{1}{x^2+y}$ $with\ y(4)=4$ compute the values of y(4.1) and y(4.2) by the Taylor's series method.

b) For the equation $\frac{dy}{dx}=x+xy^4, y(0)=3$ compute the values y(0.1) and y(0.2) by Picard's method. [5+5]

OR

11. For the initial value problem $\frac{dy}{dx}=\frac{3x+y}{x+2y}$ y(1)=1 find y(1.2) and y(1.4) by the Runge-Kutta fourth order method. [10]

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